Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Spectral Analysis Algorithms for the Laser Velocimeter: A Comparative Study"

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IT was unexpected to see a paper on the spectral analysis of randomly sampled data published in the *AIAA Journal.*[1] The subject matter is certainly very relevant to the use of LDAs in aeronautical research, and this Journal provides an appropriate vehicle for the ideas to reach the experimenter. It was encouraging to have some of our publications referenced, but we very much regret that our work seems to have been completely misunderstood. The author discusses the processing of randomly sampled signals by various methods, and he makes a number of quite incorrect assertions as to the accuracy and variability of the resulting power spectra. Indeed, there is considerable confusion in the paper between the effects of bias and variability of the spectral estimates. Rather than comment on every statement with which we disagree, and there are a great number of these, we will attempt to discuss only the main points at issue.

In our Refs. 2 and 3 we discuss the two basic estimators related to those commonly used for processing equispaced data samples:

$$\hat{S}_a(f) = \frac{2}{\nu^2 T} \sum_{k>j}^{n} \sum^{n} X(t_j)X(t_k)\cos[2\pi f(t_k - t_j)] \tag{1}$$

$$\hat{S}_b(f) = \frac{2}{\nu^2 T}\left\{\left|\sum_j^n X(t_j)e^{i2\pi f t_j}\right|^2 - \sum X^2(t_j)\right\} \tag{2}$$

These have been shown to be appropriate for signals sampled at Poisson time instants and specifically exclude situations where the sample times are dependent on the sample values.

The correlation estimator, Eq. (1), specifically excludes contributions arising from the zero-lagged products, i.e., the squares $j=k$. Similarly the direct estimator, Eq. (2), requires the subtraction of the signal mean-square from the squared Fourier coefficients. We note that the second term in Eq. (2) is missing in the formulation used by Bell, and this accounts for the appalling biased spectral estimates shown in his Fig. 2. By the use of an ingenious cross-correlation technique the bias and additional variability then arising were somewhat reduced, but such an approach is a poor substitute for the correct procedure—Eq. (2).

To demonstrate this point, Fig. 1 shows the spectral estimates obtained from a randomly sampled sine wave, using the correct direct Fourier transform method [with the variance subtracted—Eq. (2)]. For simplicity a simple "boxcar" window has been used, rather than a Hanning window as employed by Bell. Here the data parameters (signal frequency, mean sampling rate, block size, etc.) are the same as those chosen by the author; Fig. 1 can thus be compared with his Fig. 2. It is noted that the correct spectral estimators show a peak at the expected frequency which is two orders of magnitude above the estimators at the "background" frequencies (i.e., about 40 dB, compared with the author's value of 10 dB). Moreover, the estimators in the background fluctuate about a mean value of zero (hence our choice of a linear vertical scale), as predicted in Ref. 2. This is in contrast to the results shown in the author's Fig. 2, which are everywhere positive. The magnitude of the variability of the estimators shown in our Fig. 1 is entirely in accord with the theoretical prediction given in Ref. 2.

Only when the author has been able to properly demonstrate that our analysis is wrong or that our numerical simulations are at fault in some way can his comments on the accuracy of our variability predictions be taken at all seriously.

The two spectral estimators (1) and (2) both suffer from excess variability, arising from the irregular sample times, over and above that associated with periodic sampling. It is possible to consider other estimators that, in some sense, enable one to "win." In Ref. 5 we discuss a way of doing this without introducing additional information. This technique

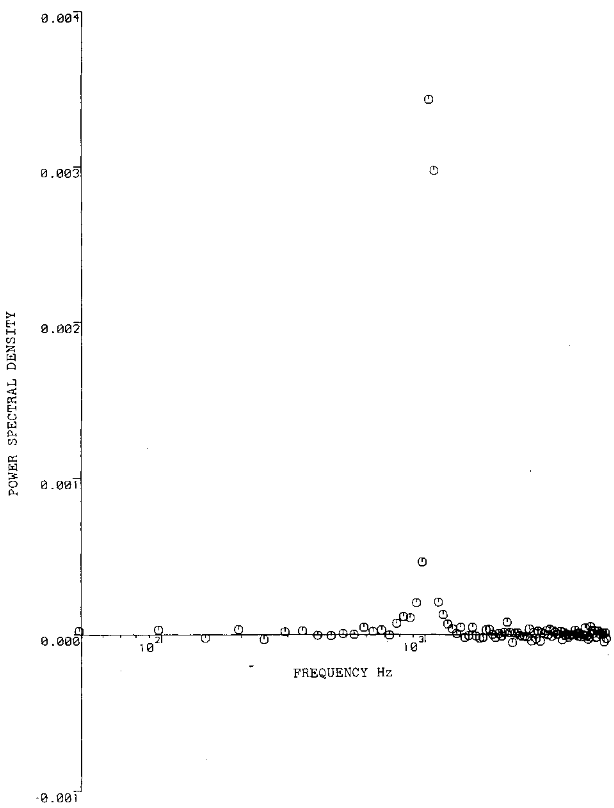

Fig. 1 Spectral estimates formed by the periodogram method. Data consisted of 2500 Poisson sampled values of a 1000 Hz sine wave generated at a mean sampling rate of 12,500/s. No Hanning window was applied.

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can show substantial gains in the sample length required to achieve a given variability. Extensive vertification of our scheme has been carried out on simulated data. In one practical but rather limited example the technique was shown to reduce the variability of the estimates by some 60% as predicted by our theory for the parameters arising in that case. We feel that Bell's dismissal of this approach is quite unfounded.

We are well aware that Eqs. (1) and (2) are not computationally very efficient, and went to some pains in Refs. 2 and 3 to discuss faster implementations of these schemes. Indeed, we have considered the slotted correlation approach and have shown by numerical experiment that the variability is similar to that predicted for Eq. (1). In fact, this has been shown to be a practical and fairly straightforward way of getting spectra from randomly acquired data, and have been involved in its use in Refs. 6 and 7. We also suggest various ways of using a look-up table or FFT method in the direct transform. It is not helpful at all to compare Eq. (2) with a slotted correlation approach insofar as speed is concerned. If speed is of extreme significance then our proposals in Ref. 8 on a rapid estimator should be examined.

References

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Reply by Author to M. Gaster and J. B. Roberts

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THE comments made by Gaster and Roberts concerning Ref. 1 provide a welcome opportunity to clarify some of the points brought out in the paper. First of all, the unbiased estimator for the power spectrum based on the periodogram is

$$\hat{S}(\omega)=\frac{T}{N^2}\left\{\left|\sum_k X_k e^{i\omega t_k}\right|^2-\sum_k X_k^2\right\} \tag{1}$$

This equation should be compared with Eq. (7) of Ref. 1. In the reference it should have been clearly stated that Eq. (7) yields a biased estimate and that the approach used in the paper was an attempt to circumvent the effects of that bias. However, as Gaster and Roberts point out and as a comparison of the results presented in their Comment with the results of Fig. 2 in the paper indicate, this perhaps ingenious approach is a poor substitute for the direct approach of Ref. 2. This, then, explains the discrepancy in the variability predicted from the rigorous analysis in Ref. 2 and that observed in the paper.

The results in the paper were obtained by taking the logarithm of the magnitude of the spectral estimate and multiplying by ten. This was done to highlight the variability of the spectrum. Of course Eq. (1) can produce negative values, but it was felt that the magnitude of the spectral estimate was most germane to the study of variability. Using Eq. (1), the peak at the expected frequency occurs 20 DB above the estimators at the "background" frequencies as shown by the results presented in the comments by Gaster and Roberts.

As Gaster and Roberts also demonstrate, the variability can be reduced by a suitable choice of sampling rate when using Eq. (1). In cases where this approach is unfeasible, the author has found in recent research that the relatively simple technique described in Ref. 3 can be used to decrease the variability for certain spectra.

The author shares the commentors' opinion that the subject matter of the paper is very relevant to data analysis with the laser velocimeter and hopes that further research into this important field is forthcoming.

References

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Comment on "Generalized Coordinate Forms of Governing Fluid Equations and Associated Geometrically Induced Errors"

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IN a recent paper,[1] Hindman presented an analysis regarding geometrically induced errors arising from conservation laws written in a generalized coordinate system. Specifically, the observation is made that if a numerical integration algorithm is consistently formulated, it should be able to integrate a constant uniform flow solution without producing any changes. An analysis is performed by Hindman on two-dimensional chain rule, weak, and strong conservation law forms (CRCLF, WCLF, and SCLF, respec-

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